Spin Dynamics and Snake Resonances

Thomas Roser
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Spin Dynamics in Rings

Precession Equation in Laboratory Frame:
(Thomas [1927], Bargmann, Michel, Telegdi [1959])

$$d\mathbf{S}/dt = -(e/\gamma m)\,[(1+G\gamma)\mathbf{B}_{\perp} + (1+G)\,\mathbf{B}_{||}] \times \mathbf{S}$$

Lorentz Force equation:

$$d\mathbf{v}/dt = -(e/\gamma m)\,[\mathbf{B}_{\perp}] \times \mathbf{v}$$

- For pure vertical field:
  Spin rotates $G\gamma$ times faster than motion, $\nu_{sp} = G\gamma$
- For spin manipulation:
  At low energy, use longitudinal fields
  At high energy, use transverse fields

Spin tune and Depolarizing Resonances

Depolarizing resonance condition:

Number of spin rotations per turn = Number of spin kicks per turn

Imperfection resonance (magnet errors and misalignments):

$\nu_{sp} = n$

Intrinsic resonance (Vertical focusing fields):

$\nu_{sp} \pm Q_y = Pn$

P: Superperiodicity [AGS: 12]

Q_y : Betatron tune [AGS: 8.75]

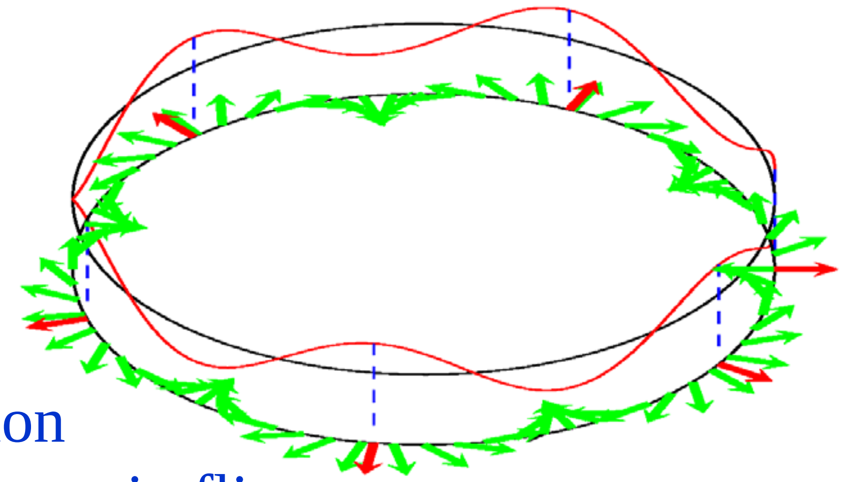

Illustration by W.W. MacKay

Weak resonances: some depolarization

Strong resonances: partial or complete spin flip

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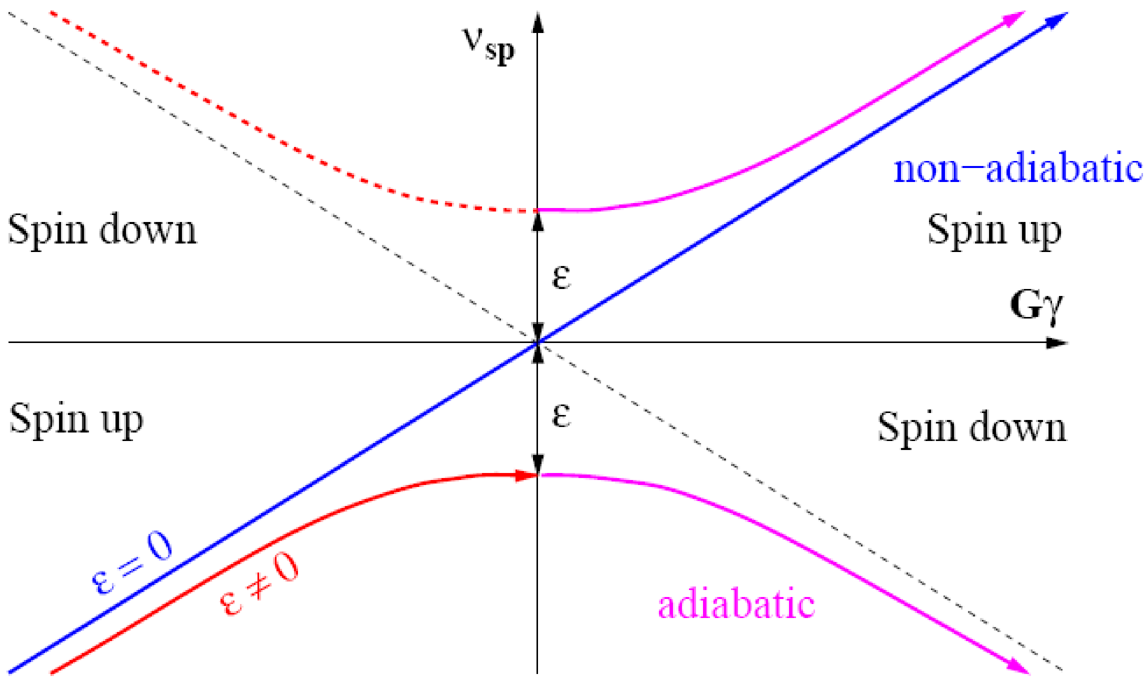
Resonance Crossing at Intermediate Energies

Froissart-Stora: $\frac{P_f}{P_i} = 2\,e^{-\frac{\pi\varepsilon^2}{2\alpha}} - 1$ [α: crossing speed]

Non-adiabatic ($\varepsilon^2/\alpha \ll 1$) ↔ Adiabatic ($\varepsilon^2/\alpha \gg 1$)

$P_f/P_i = 1$ $\qquad$ $P_f/P_i = -1$



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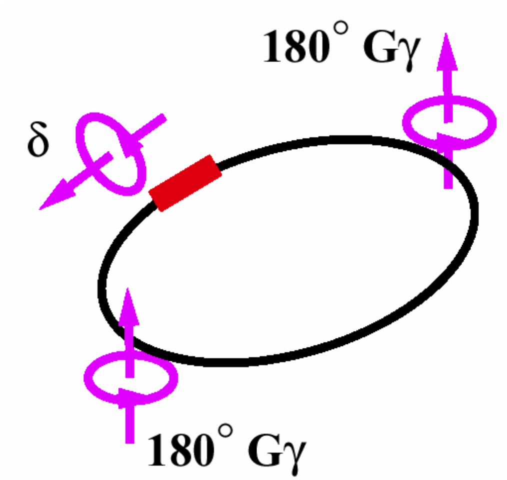
Siberian Snakes (Local Spin Rotators)

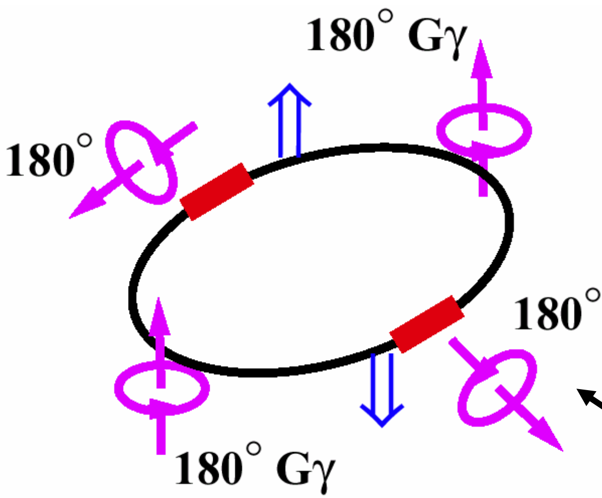


$$\boxed{\cos(180^\circ\,\nu_{sp}) = \cos(\delta/2)\cdot\cos(180^\circ\,G\gamma)}$$

$\delta \neq 0^\circ \rightarrow \nu_{sp} \neq n$

<u>No</u> imperfection resonances

Partial Siberian snake (AGS)



$\delta = 180^\circ \rightarrow \nu_{sp} = ½$

<u>No</u> imperfection resonances <u>and</u>

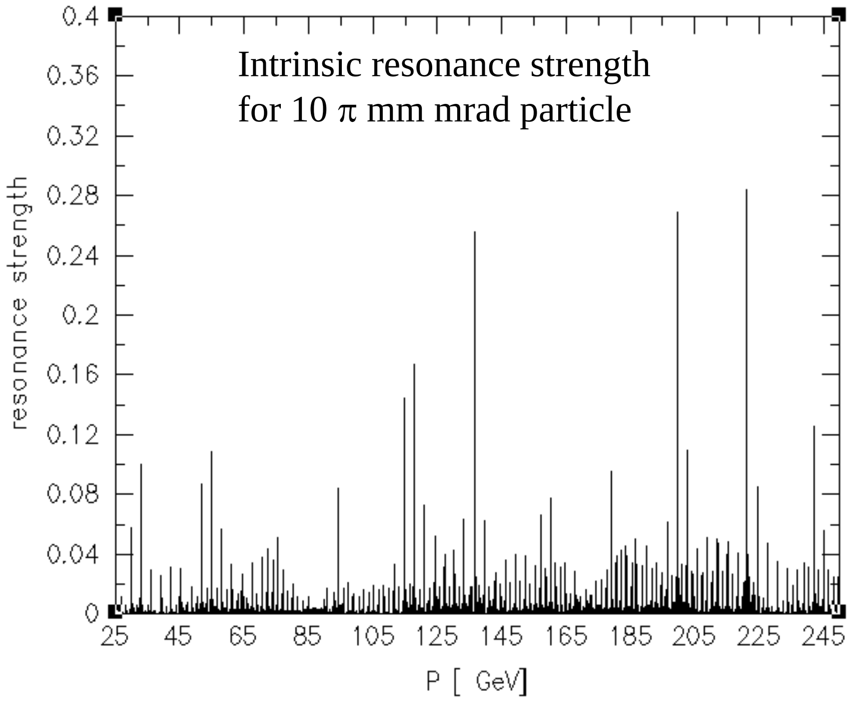
<u>No</u> Intrinsic resonances

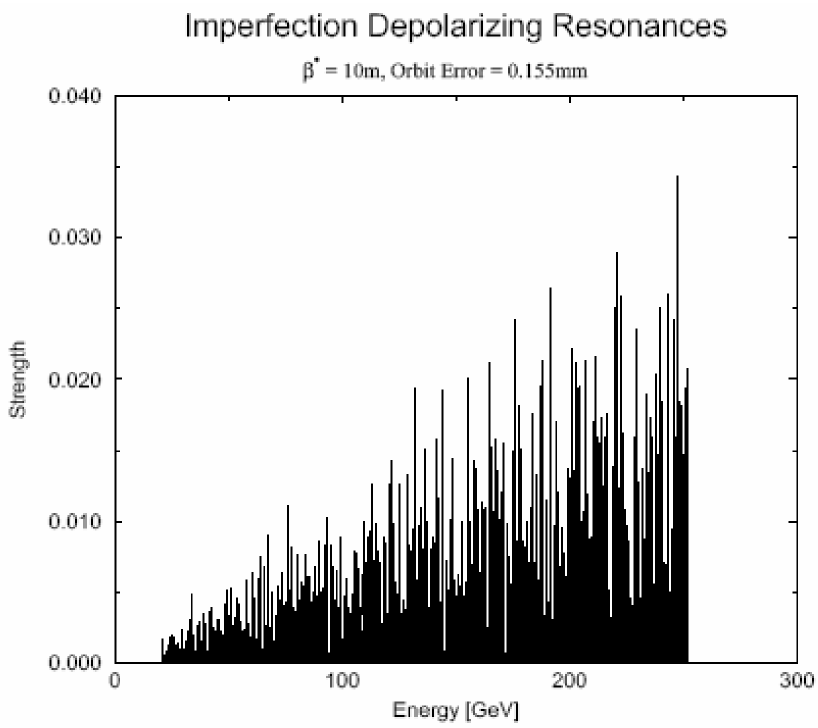
Full Siberian Snake

(Ya.S. Derbenev and A.M. Kondratenko)

Two Siberian Snakes in RHIC

Spin Resonances in RHIC





(Naïve) Limits for Siberian Snakes

Spin rotation of Siberian snake (δ) > Spin rotation of driving fields (ε)

"Spin rotation of Siberian snake drives strong imperfection resonance"

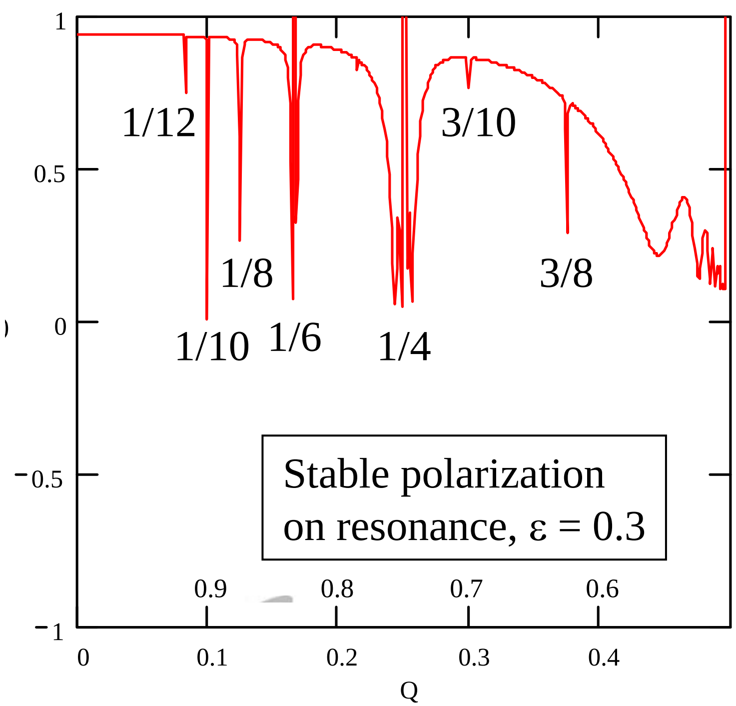
Imperfection resonances	**$\varepsilon \propto$ Energy**
Intrinsic resonances	**$\varepsilon \propto \sqrt{\text{Energy}}$**

Partial Siberian snake (AGS, $\delta = 9°$)	**$\varepsilon < \delta/360°$**
One full snake	**$\varepsilon < 1/2$**
Two full snakes (RHIC)	**$\varepsilon < 1$**
N full snakes (LHC? $N \approx 16$)	**$\varepsilon < N/2$**

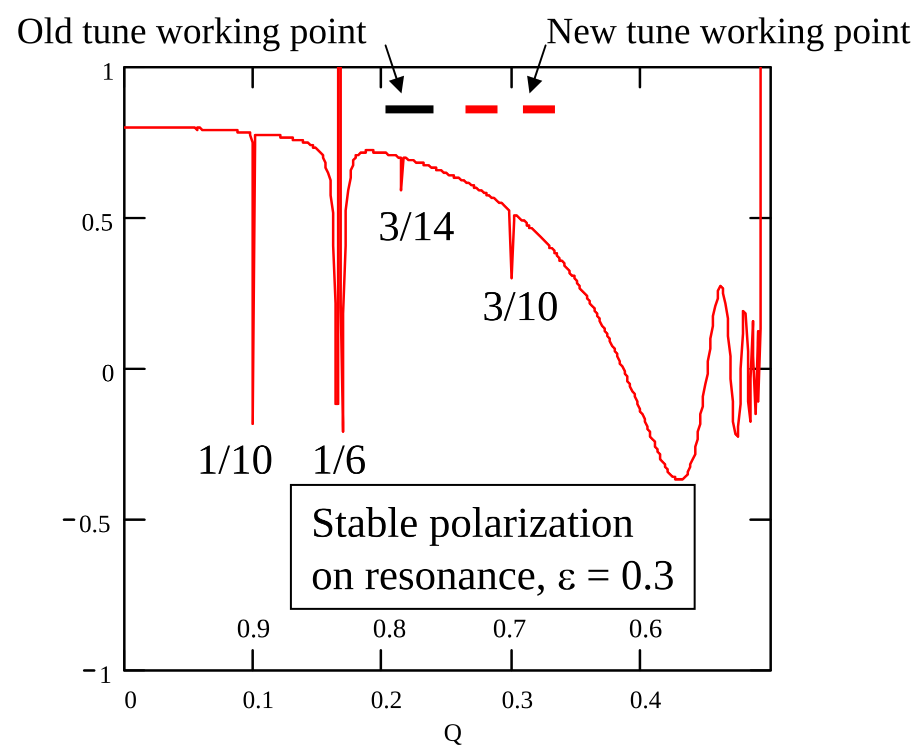
Polarization with Snakes – Snake Resonances

- Higher order resonance condition $\nu_{sp} + mQ_y = k$ (m, k = integer) driven by interaction of intrinsic resonance $G\gamma + Q_y = k$ with large spin rotations of dipoles and snakes.
- No non-linear drive term necessary – combination of rotations is already non-linear.
- “Snake resonance strength” depends on intrinsic resonance strength and therefore energy
- For $\nu_{sp}=1/2+\Delta\nu_{sp} \rightarrow Q_y = (2k-1)/2m-\Delta\nu_{sp}/m$
- First analytical solution of isolated resonance with snakes by S.R. Mane, NIM A 498 (2003) 1

single snake
or two snakes with orbit errors



two snakes **(m: odd)**



Slow Crossing of a Single Resonance with and w/o Snakes

Without snakes: spin flip, width ~ ± 5ε

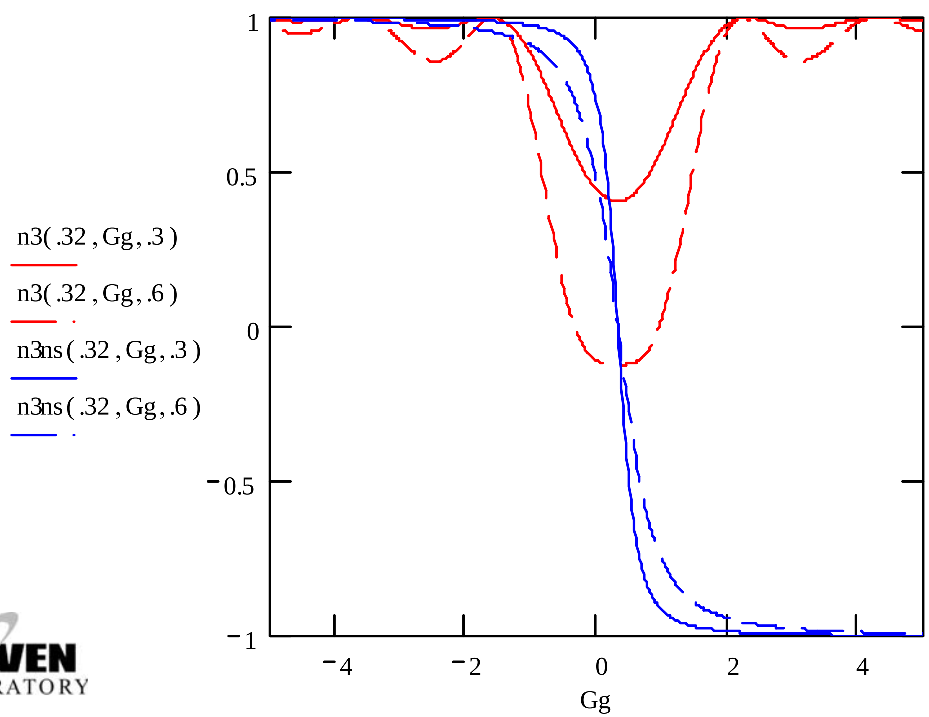
With snakes: opening/closing of "spin cone", nodes at ± 2

$$\Omega(Q,Gg) := \sqrt{(Q-Gg)^2+\varepsilon^2} \qquad \eta(Q,Gg) := \frac{\varepsilon}{\Omega(Q,Gg)}\cdot\sin\left(\frac{\Omega(Q,Gg)\cdot\pi}{2}\right)$$

$$S(m,\delta) := \prod_{k=1}^{m}\sin(k\cdot\pi\ \delta) \qquad C(m,\delta) := \prod_{k=0}^{\mathrm{trunc}(m)}\cos((m-k)\ \pi\ \delta)$$

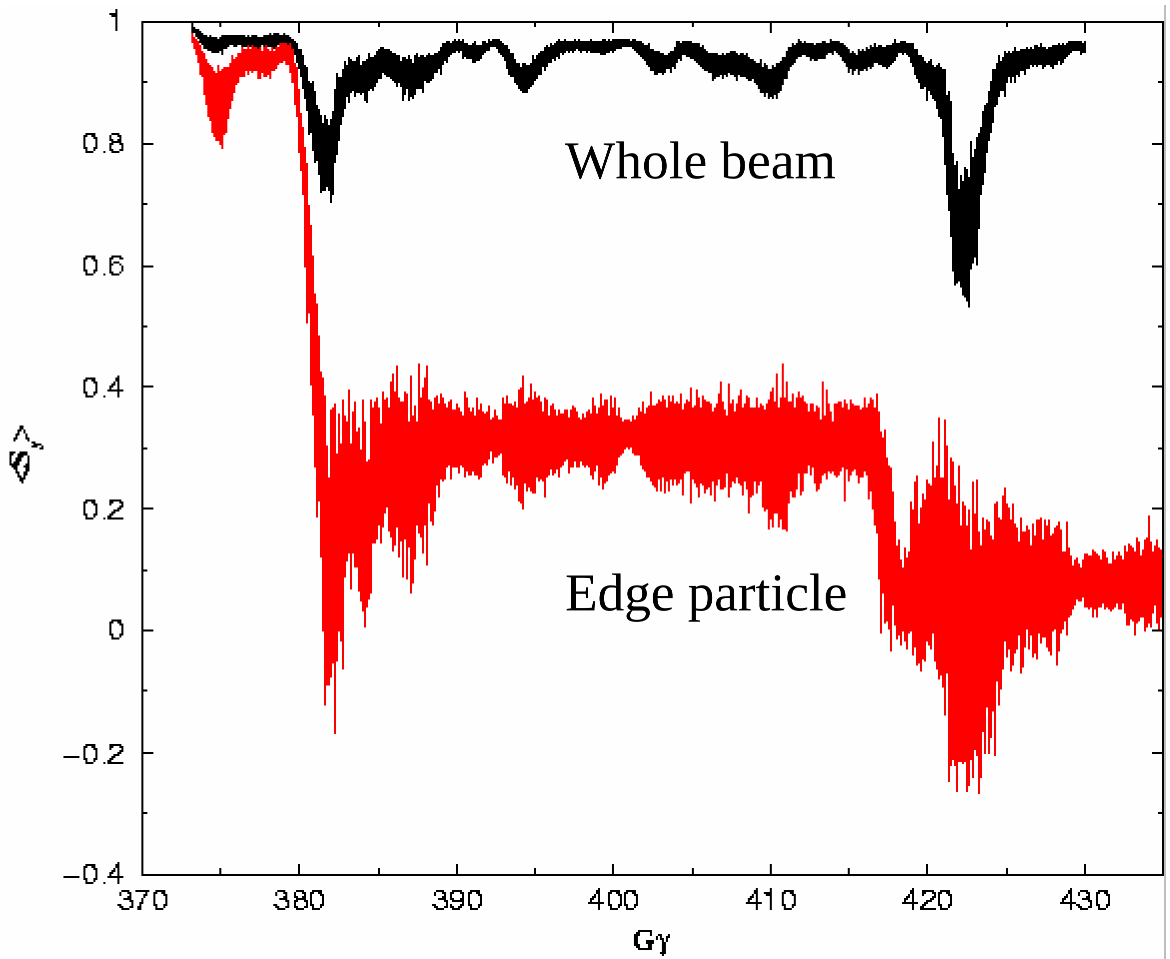
$$n3(Q,Gg) := \left(1-\eta(Q,Gg)^2\right)\left[1+\sum_{k=1}^{40}(-1)^k\cdot\frac{C\left(k,Q-\frac{1}{2}\right)^2}{S\left(k,Q-\frac{1}{2}\right)^2}\cdot\eta(Q,Gg)^{2\cdot k}\right]$$

$$n3ns(Q,Gg) := \frac{(Q-Gg)}{\sqrt{(Q-Gg)^2+\varepsilon^2}}$$



S.R. Mane, NIM A 498 (2003) 1

Spin tracking trough strongest RHIC resonances



- Two Siberian snakes
- 1 mm rms misalignment
- 0.2 mm rms corrected closed orbit
- 20 π μm emittance (95%)

[A. Luccio et al. (SPINK)]